Measuring Mismatch in the Swedish Labour Market

Georg Marthin
Finanspolitiska rådet

The views expressed in this report are those of the authors and do not necessarily represent those of the Swedish Fiscal Policy Council. I am grateful for comments from Aron Berg, Niklas Frank, Peter Fredriksson, Max Elger, John Hassler, Erik Höglin, Karine Raoufinia, Göran Selin and Joakim Sonnegård.
Finanspolitiska rådet är en myndighet som har till uppgift att göra en oberoende granskning av regeringens finanspolitik. Rådets uppgifter fullföljs framför allt genom publiceringen av rapporten Svensk finanspolitik som lämnas till regeringen en gång per år. Rapporten ska kunna användas som ett underlag bland annat för riksdagens granskning av regeringens politik. Som ett led i uppdraget anordnar rådet även konferenser och utger skrifter om olika aspekter på finanspolitiken. I serien Studier i finanspolitik publiceras fördjupade studier eller rapporter som härrör från externa uppdrag.

Finanspolitiska rådet
Box 3273
SE-103 65 Stockholm
Kungsgatan 12-14
Tel: 08-453 59 90
Fax: 08-453 59 64
info@finanspolitiskaradet.se
www.finanspolitiskaradet.se

ISSN 1654-8000
Abstract
In this paper we create a measurement for labour market mismatch, which we define as the difference between a more efficient allocation of job-seekers and the actual distribution of job-seekers. This is done by deriving an allocation rule from a dynamic search model where a social planner chooses the optimal allocation of job-seekers. This rule is then used as a benchmark from which we create an index for geographical and occupational mismatch. We show that there is geographical mismatch that could explain up to 0.3 percentage points of the unemployment. We also measure a seemingly large occupational mismatch in the Swedish labour market which could explain up to 1.8 per cent of the unemployment. Through studying its trend and different levels of disaggregation we conclude that occupational mismatch is changing in its composition and that mismatch might be an increasing problem. Furthering our study of occupational mismatch we note that the mismatch index is less for the inexperienced job-seekers at the Swedish Public Employment Service than for the experienced.
1 Introduction

There are many frictions between vacancies and job-seekers. Do job-seekers have the right set of skills? Do they have the right education? Are there moving costs? Will they even know about the vacancy? The possible reasons to why one vacancy and one job-seeker do not result in one match are almost infinite. The aim of this paper is not to explain why there are inefficiencies but instead attempt to measure the extent of existing mismatch. This will be done across certain dimensions, namely geography and occupation. By doing so we can provide insight into where the problem with mismatch is more severe. By creating a measurement for mismatch that can be studied over time we will also be able to study the dynamics of labour market mismatch – is the issue increasing in severity or have recent labour market developments decreased the problem?

This paper’s contribution is to construct a mismatch index, based on data from the Swedish Public Employment Service, which fits the proposed model and its assumptions to a high degree, lessening the need for corrections, adjustments and further assumptions when fitting the data to the model.

The method used is one developed by Sahin et al (2011). From a dynamic search model they propose a planner’s problem, the solution of which provides us with an allocation rule for the job-seekers over sectors. This rule is then used to create an index measuring the share of matches lost between actual observed data and the planner’s optimal allocation across different types of sectors. To give some perspective of the gains to be made from a more efficient allocation, a counterfactual unemployment rate is constructed. It should also be noted that our measurement is an upper bound of mismatch since we do not take sector reallocation costs into account, such as moving cost or education costs. Given these reallocation costs the difference between optimal and actual unemployment distribution should be less since there ought to be situations where reallocation is not worth the cost.

The first part of this paper is a short depiction of the methodological background of the mismatch index. In the next part, the model is explained and the mismatch index is derived. A description of the data and the empirical methods used to estimate the parameters will then follow. After these estimations, the empirical results will be presented and some possible interpretations will be discussed. Finally, comparisons are made with similar studies in Sweden and elsewhere.

2 Background

The conceptual foundation of Sahin et al. (2011) can be found in a paper by Jackman and Roper (1987) where they, from a static model, derive three mismatch indices. The first measures the number of unemployed allocated in the wrong sector relative to the labour force. The second measures the number of unemployed allocated in the wrong sector relative to the number of unemployed in the whole economy. Lastly, the third measures the amount of
matches “lost” compared to the optimal distribution. Jackman and Roper argue that an efficient allocation of unemployed would be when labour market tightness, the relationship between vacancies and unemployed, is equal in all sectors. This notion is kept in the framework of the method used in this paper, and if all sectors were homogeneous we would get the same results.

Having said that, an important part of the methods used in this paper is the discrepancies between sectors. A recent study demonstrating this is Barnichon and Figura (2011). They show that there are definite incongruities across sectors and that the dynamics of the aggregated matching efficiency can be better explained by assuming that the different sectors have different job-finding probabilities.

3 Creating a measurement

The basic concept behind the mismatch index that Sahin et al. (2011) have defined is that we compare the actual unemployment distribution to a social planner’s optimal allocation of the unemployed. To create the planner’s optimal allocation we must first stipulate the economic environment in which the planner acts. This environment will give the planner the option to choose where to allocate the unemployed with the goal to maximise aggregate utility. The solution to this problem will give us an allocation rule, a benchmark, on which we base our index. In order to clarify what our index actually implies, our last step will be to create an expression for the counterfactual unemployment, i.e. an unemployment rate where there is no mismatch between the studied sectors.

3.1 The Planners problem

We start by defining the variables and basic attributes of the model.

The main concept of this paper is dividing the labour market into different sectors. These sectors we denote $i$. Vacancies in sector $i$ ($v_i$) are treated exogenously and can be seen as possible increases in production. The employed in sector $i$ ($e_i$) are the ones who produce within each sector. The unemployed in sector $i$ ($u_i$) are the only ones looking for jobs, i.e. the already employed are not looking for new jobs and people outside of the labour market are not seen as job-seekers. The labour force is defined as $l = \Sigma_{i=1}^l e_i + u_i$ where $l \leq 1$. If the labour force is less than one this implies that there are people outside of the labour force. Being outside of the labour force will give zero utility. The cost of searching for a job ($\xi$) is treated as a negative utility. Matches in sector $i$ ($h_i$) between vacancies and unemployed are governed by the function $\Phi\phi_i m(u_i, v_i)$ where $m$ is strictly increasing and concave in both arguments and homogeneous of degree one (constant returns to scale). In sector $i$ the matching efficiency will be defined by $\Phi\phi_i$ where $\phi_i$ denotes sector $i$’s specific matching efficiency and $\Phi$ is the whole economy’s matching efficiency. Much like matching efficiency productivity is defined as $Z\phi_i$ where $\phi_i$ denotes sector $i$’s specific productivity and $Z$ is the whole economy’s productivity. The productivity share of a newly matched person
compared to an already employed person (γ) captures the learning cost by always being less than one. Combining productivity with the employed we get an expression for the output in each sector, \( Z Z_t (e_l + \gamma h_t) \). Finally, matches can be destroyed. This is expressed with \( \delta \) which describes the fraction of all matches destroyed.

**The model dynamics**

First of all, time is discrete. The changes, or shocks, in \( Z, \delta, \Phi \) are described by the conditional distribution function \( \Gamma_{Z,\delta,\Phi}(Z', \delta', \Phi'; Z, \delta, \Phi) \) implying that the next period’s \( Z, \delta, \Phi \), denoted with a prime, solely depends on their present state. The changes in the vector of vacancies are described by \( \Gamma_{v}(v'; v, Z', \delta', \Phi') \), which shows that future vacancies not only depend on its current state but is correlated with future \( Z, \delta, \Phi \).

The vector for sector-specific matching efficiencies is drawn from the independent distribution \( \Gamma_{\phi}(\phi'; \phi) \) and the vector for sector-specific productivity from \( \Gamma_{z}(z'; z) \) which, contrary to the process of \( \phi \), is assumed to have a linear conditional mean function which makes it possible to describe estimated productivity as \( \mathbb{E}(\varepsilon_l) = \rho z_t \).

At the start of each period the aggregated shocks in \( Z, \delta, \Phi \), and the distributions of \( \phi_i, z_i, v_i, e_i \) across sectors are observed. The size of the labour force is determined in the previous period and is thus seen as given in this period. Since the labour force and the number of people employed are given, we also know the total amount of unemployed in the economy. At this point the planner makes a decision regarding the distribution of the unemployed across sectors. When the planner has made a choice regarding the allocation, the matches are made according to the matching function \( h_t = \Phi \phi_i m(u_i, v_i) \). Hereafter there is production in existing matches \( e_l \) and the new matches \( h_t \). A share of the matches, \( \delta \), are destroyed which defines the vector \( \{e_l'\} \). Finally, the planner decides on the size of the labour force in the next period which will determine the stock of unemployed in the coming period.

**The planner’s solution**

As stated above, the planner’s task is to, as efficiently as possible, allocate the unemployed and decide the size of the next period’s labour force which is done by maximising the following function where vacancies, matching efficiencies, productivity and the destruction rate of matches are given. We describe the problem in recursive form which is practical when solving the problem for any point in time.

\[
V(e, u; v, \phi, z, Z, \delta, \Phi) = \max_{\{u_i, l'\}} \sum_{i=1} Z Z_i (e_i + \gamma h_i) - \xi u + \beta \mathbb{E}[V(e', u'; v', \phi', z', Z', \delta', \Phi')]
\]
Subject to

\[ \sum_{i=1}^{I} u_i \leq u \]  
(1)

\[ h_i = \Phi \phi_i m(u_i, v_i) \]  
(2)

\[ e'_i = (1 - \delta)(e_i + h_i) \]  
(3)

\[ u' = l' - \sum_{i=1}^{I} e'_i \]  
(4)

\[ u_i \in [0, u] \quad l' \in [0, 1] \]  
(5)

\[ \Gamma_{Z, \delta, \phi}(Z', \delta', \Phi'; Z, \delta, \Phi), \Gamma_{\psi}(\psi; v, Z', \delta', \Phi'), \Gamma_{\phi}(\phi; \phi), \Gamma_{Z}(Z'; \psi) \]  
(6)

The function states that the planner’s utility in this period is the sum of production in each sector minus the search cost and plus the discounted utility of the subsequent period. The first restriction (1) states that the planner has \( u \) unemployed to allocate across sectors. The second (2) that \( \Phi \phi_i m(u_i, v_i) \) new matches are made on top of the already existing ones. The third (3) describes the coming periods employed. The fourth (4) expresses the next period’s stock of unemployed. The fifth (5) defines the range of \( u_i \) and \( l' \). And lastly, the sixth (6) describes the stochastic processes that are seen as exogenously given.

Here, only a short recapitulation of the solution will be given. For a more detailed solution of the problem, see Appendix A.

The following is derived from the optimisation problem.

\[ z_i \phi_i m(u_i) \left( \frac{v_i}{u_i} \right) = \frac{\mu}{(Z\gamma \phi + (1 - \delta)\phi \beta \rho \mathbb{E}[\psi(Z', \delta', \Phi')])} \]  
(7)

The right hand side is independent of sector implying that the optimal distribution provides us with a relationship between productivity, matching efficiency and the ratio between vacancies and unemployed which states that this relationship must be the same over all sectors when optimising the unemployment distribution. The planner’s allocation rule can be expressed as

\[ z_i \phi_i m(u_i) \left( \frac{v_i}{u_i} \right) = \cdots = z_i \phi_i m(u_i) \left( \frac{v_i}{u_i} \right) = \cdots = z_i \phi_i m(u_i) \left( \frac{v_i}{u_i} \right) \]  
(8)

A larger (smaller) ratio between vacancies and unemployed, a higher (lower) matching efficiency and a higher (lower) productivity, suggests that there should be more (less) unemployed people in that sector.
If we define our matching function $m(u_i, v_i)$ as a Cobb-Douglas function

$$m(v_i, u_i) = v_i^\alpha u_i^{1-\alpha}$$  \hspace{1cm} (9)

Where $\alpha \in (0,1)$. We plug equation (9) into (8) and solve for $u_{it}^*$

$$u_{it}^* = (z_{it} \phi_i)^{\frac{1}{\alpha \beta}} \frac{v_{it}}{\sum_{i=1}^{I} (v_{it} (z_{it} \phi_i)^{\frac{1}{\alpha \beta}})} u_t$$  \hspace{1cm} (10)

We now have an expression for the most efficient amount of job-seekers in sector $i$. This is the integral part in our mismatch index, which the following section will show.

3.2 The Mismatch Index

In an attempt to create an intuition regarding the index we start off by assuming that $z_i$ and $\phi_i$ are equal across sectors. This implies that the ratio of vacancies and unemployment should be equal across sectors for an optimal solution. The total amount of matches/hires at time $t$ is

$$h_t = \Phi_t v_t^\alpha u_t^{1-\alpha} \left[ \sum_{i=1}^{I} \left( \frac{v_{it}}{v_t} \right)^\alpha \left( \frac{u_{it}}{u_t} \right)^{1-\alpha} \right]$$

and the optimal amount of hires is

$$h_t^* = \Phi_t v_t^\alpha u_t^{1-\alpha}$$

In other words $1 - \sum_{i=1}^{I} \left( \frac{v_{it}}{v_t} \right)^\alpha \left( \frac{u_{it}}{u_t} \right)^{1-\alpha}$ describes the fraction of hires lost due to asymmetric distribution of vacancies and unemployment. Formally we describe our index as

$$M_t^h = 1 - \frac{h_t}{h_t^*} = 1 - \frac{\Phi_t v_t^\alpha u_t^{1-\alpha} \left[ \sum_{i=1}^{I} \left( \frac{v_{it}}{v_t} \right)^\alpha \left( \frac{u_{it}}{u_t} \right)^{1-\alpha} \right]}{\Phi_t v_t^\alpha u_t^{1-\alpha}}$$

$$M_t^h = 1 - \sum_{i=1}^{I} \left( \frac{v_{it}}{v_t} \right)^\alpha \left( \frac{u_{it}}{u_t} \right)^{1-\alpha}$$  \hspace{1cm} (11)

It is also possible to express the share of unemployed searching in the “wrong” sector. The amount of unemployed searching in the wrong sector is

$$u_t^M = \frac{1}{2} \sum_{i=1}^{I} \left| u_{it} - u_{it}^* \right| = \frac{1}{2} \sum_{i=1}^{I} \left| \frac{u_{it}}{u_t} - \frac{v_{it}}{v_t} \right| u_t$$
and taking this relative to the actual unemployment we get the share

\[ \frac{u_t^M}{u_t} = \frac{1}{2} \sum_{i=1}^I \left| \frac{u_{it}}{u_t} - \frac{v_{it}}{v_t} \right| \]  

(12)

This expression is in itself a measure of mismatch, but it does not actually give a number of how many more job-seekers that will be employed if reallocated, as the first index does, and thus it lacks in policy relevance. However, it is interesting as a complement to the first index as it states how many job-seekers should be reallocated to achieve the optimal number of hires implied by the first index, and thus it can be seen as somewhat of a proxy for the cost of reallocation.

Heterogeneity across sectors

In our solution of the planner’s problem we assumed heterogeneity across sectors and this must of course be accounted for in our index.  

\[ h_t^* = \Phi_t v_t^\alpha u_t^{1-\alpha} \left[ \sum_{i=1}^I \phi_t \left( \frac{v_{it}}{v_t} \right)^\alpha \left( \frac{u_{it}}{u_t} \right)^{1-\alpha} \right] \]  

(13)

which together with equation (10) yields

\[ h_t^* = \Phi_t v_t^\alpha u_t^{1-\alpha} \Phi_{xt} \]

where

\[ \Phi_{xt} = \bar{x}_t \frac{\sum_{i=1}^I \left( \frac{1}{x_{it}} \right) x_{it} \frac{1}{x_t} \left( \frac{v_{it}}{v_t} \right)}{\sum_{i=1}^I \frac{1}{x_{it}} \left( \frac{v_{it}}{v_t} \right)} \]

and

\[ \bar{x}_t = \left( \sum_{i=1}^I \frac{1}{x_{it}} \left( \frac{v_{it}}{v_t} \right) \right)^\alpha \]

Actual hires are given by

\[ h_t = \Phi_t v_t^\alpha u_t^{1-\alpha} \left[ \sum_{i=1}^I \phi_t \left( \frac{v_{it}}{v_t} \right)^\alpha \left( \frac{u_{it}}{u_t} \right)^{1-\alpha} \right] \]

---

1 For a more detailed derivation of the index, see Appendix B.
2 To simplify our notation we will from now on write \( z_{it} \Phi \) which is a measure of the total matching efficiency in sector \( i \) at time \( t \).
The index with heterogeneity can thus be written as

\[ M_{ht} = 1 - \frac{h_t}{\bar{h}_t} = 1 - \frac{\Phi_t v_t^a u_t^{1-a} \left[ \sum_{i=1}^{l} \phi_i \left( \frac{v_{it}}{v_t} \right)^a \left( \frac{u_{it}}{u_t} \right)^{1-a} \right]}{\Phi_t v_t^a u_t^{1-a} \phi_{xt}} \]

\[ = 1 - \sum_{i=1}^{l} \left( \frac{\phi_i}{\phi_{xt}} \right) \left( \frac{v_{it}}{v_t} \right)^a \left( \frac{u_{it}}{u_t} \right)^{1-a} \] (14)

In this heterogeneous case the number of misallocated job-seekers is

\[ u^M_t = \frac{1}{2} \sum_{i=1}^{l} |u_{it} - u_{it}'| \]

This together with equation (10) gives us the following expression

\[ \frac{u^M_t}{u_t} = \frac{1}{2} \sum_{i=1}^{l} \frac{|u_{it} - u_{it}'|}{u_t} - \left( \frac{x_{it}}{x_t} \right)^{1-a} \left( \frac{\bar{v}_{it}}{v_t} \right) \] (15)

3.3 Properties of the index

A main attribute of our index is that it is increasing with the order of disaggregation of sectors. If we assume that the sectors \( I \) can be decomposed into subsectors \( J \) we can prove that the index \( M^I_t \) will always be smaller than \( M^I_{jj} \). The first step in doing this is rearranging equation (11) so that we separate the sector-specific vacancies and unemployed from the total versions of these variables.

\[ 1 - M_t^I = \frac{1}{\nu^a u^{1-a}} \sum_{i=1}^{l} v_{it}^a u_{it}^{1-a} \]

Implementing the measure on a subsector level yields

\[ 1 - M^I_{jj} = \frac{1}{\nu^a u^{1-a}} \sum_{i=1}^{l} \left( \sum_{j=1}^{j} v_{ij} \right)^a \left( \sum_{j=1}^{j} u_{ij} \right)^{1-a} \]

Comparing this to our more disaggregated index

\[ 1 - M^I_{jj} = \frac{1}{\nu^a u^{1-a}} \sum_{i=1}^{l} \sum_{j=1}^{j} v_{ij}^a u_{ij}^{1-a} \]
it becomes evident, as \( \alpha \in (0,1) \) and following Hölder’s inequality, that

\[
\left( \sum_{j=1}^{J} v_{ij} \right)^{\alpha} > \left( \sum_{j=1}^{J} v_{ij}^{\alpha} \right)
\]

and that

\[
\left( \sum_{j=1}^{J} u_{ij} \right)^{1-\alpha} > \left( \sum_{j=1}^{J} u_{ij}^{1-\alpha} \right)
\]

thus proving that

\[
1 - M_{ij}^{\alpha} = \frac{1}{\nu^{\alpha} u^{1-\alpha}} \sum_{i=1}^{I} \left( \sum_{j=1}^{J} v_{ij} \right)^{\alpha} \left( \sum_{j=1}^{J} u_{ij} \right)^{1-\alpha} > \frac{1}{\nu^{\alpha} u^{1-\alpha}} \sum_{i=1}^{I} \sum_{j=1}^{J} v_{ij}^{\alpha} u_{ij}^{1-\alpha} = 1 - M_{ij}^{\alpha}
\]

This proof holds true as long as the level of aggregation does not affect \( \alpha \), and in the heterogeneous index the sector-specific matching efficiency, \( \phi_i \), as these disaggregated variables are not additive parts of their more aggregated versions.

As the index is increasing in its level of disaggregation one should always take note of the number of sectors when comparing different dimensions or different labour markets. To appropriately compare different dimensions or markets, the number of sectors needs to be as equal as possible. An upside with this property is that we get a sense of how the mismatch is composed within a dimension. If, for example, there is very little difference between the mismatch in \( I \) and \( J \), the issues with mismatch are primarily between the sectors in \( I \) and thus to some extent pinpointing where the frictions are.

### 3.4 Counterfactual unemployment

Since we have described the share of matches lost we can use this index to derive a more efficient job-finding rate.\(^1\) This, in turn, can be used to calculate a counterfactual unemployment rate. This is only done so as to give a sense of the severity of the estimated inefficiency loss.

**Job-finding rate:**

\[
f_t = \frac{h_{i.t}}{u_{i.t}} = (1 - M_{i.t}) \Phi_t \left( \frac{v_{i.t}}{u_{i.t}} \right)^{\alpha}
\]

**Optimal job-finding rate:**

\[
f^*_t = \frac{h_{i.t}}{u_{i.t}} \Phi \left( \frac{v_{i.t}}{u_{i.t}} \right)^{\alpha} = f_t \frac{1}{(1-M_{i.t})} \left( \frac{u_{i.t}}{u_{i.t}} \right)^{\alpha}
\]

By using the definition of job-finding rate and separation rate devised in Shimer (2005) we can construct outflows and inflows from unemployment, which we can then adjust with our index.

---

\(^1\) Job-finding rate is the word used in Sahin et al. (2011). As they interpret it, it is synonymous with outflow from unemployment.
Shimer defines job-finding rate as

$$f_t = 1 - \frac{u_{t+1} - u_t^s}{u_t}$$

where $u_t^s$ is the short term unemployment, defined in our data as the number of people unemployed for less than a month. The separation rate from employment is

$$s_t = \frac{u_t^s + 1}{e_t}$$

Shimer adds $f_t$ into the separation rate, which is something we opt not to do. Shimer’s reasoning is that the monthly surveys in America, which are the basis for his analysis, will miss those who become unemployed after the first survey and get reemployed before the next survey. This error is directly dependent on the size of the outflow from unemployment. However, to correct for the fact that the unemployment spell could have started at any given point within this month, and the expected value is half a month, Shimer adjusts $f_t$ by dividing it by two. For Swedish data this is somewhat adjusted as Statistics Sweden estimates the unemployment time down to one week, thus lessening the potential underestimation of the separation rate.

Taking the separation rate and the number of vacancies as given, we adjust the job-finding rate to our optimal rate. From a given initial value of unemployment, one can then iterate forward the counterfactual rate and thus obtain a counterfactual unemployment:

$$u_{t+1} = s_t + (1 - s_t - f_t)u_t^s$$

The conclusions drawn from this counterfactual unemployment must however be modest since both vacancies and separation rates are given, and all other dynamic effects that the increased hiring should have on unemployment are not taken into account. Further, we should note that within the model, the planner could affect the size of the labour force and thus the separation rate, according to the definition above, which implies that the unemployment rate we obtain from the iteration is not the planner’s optimal unemployment rate.

### 3.5 Some caveats of the index

It cannot be stressed enough that the index does not in itself provide any evidence on why there are matching inefficiencies. It is created and used so as to catch all inefficiencies, some of which, when implemented in a real world analysis, are impossible to solve for. The most noticeable issue lies within the assumption of homogeneous job-seekers. If we imagine a market where the only friction is due to the job-seekers’ different inherent abilities, for example ambition, we would pick this up in our index and note that some sectors,

---

4 Note here that the more efficient counterfactual unemployment, $u_t^*$, is not the same as the one in the index.
where the more ambitious job-seekers are, have higher matching efficiencies and vice versa. The index would suggest a solution where more job-seekers were allocated to the ambitious labour markets with higher matching efficiency, but this would not be a more efficient solution as it only reallocates the problem without increasing the efficiency. As the problem with heterogeneous job-seekers varies in severity depending on which dimension we look at, we must keep this in mind when we interpret our results. It also hinders us from seeing our more efficient allocation as a first best solution to the planner’s problem.

4 Data and empirical method

The main contribution of this paper is combining the methods developed in Sahin et al. (2011) with data from the Swedish labour market. This provides us with both new insights in the Swedish labour market, and a test of the method itself with data that fits the model and its assumptions startlingly well.

4.1 The Swedish Public Employment Service

The Swedish Public Employment Service keeps track of all registered job-seekers and available vacancies that they are notified of. The raw data is daily and there are hundreds of different codes and attributes that one can use to sort out whatever data is needed.

As we do not have full access to the data provided by the Public Employment Service, we are in this study constrained to using what has been made available to us. It is, however, of interest that there is more detailed data kept by the Public Employment Service, if future work is to be done using the methods described in this paper.

All data attained from the Public Employment Service is monthly. Vacancies and job-seekers are described as both the stock and the inflow that month. Defining a dimension is done by sorting both vacancies and job-seekers into the same sectors. The geographical dimension is represented by the 21 Swedish counties. The occupational dimension is represented by the occupational code SSYK, which is very similar to the internationally used ISCO definition.

SSYK has four levels where the first level consists of nine different occupational areas. An example would be occupational area 3, which can be translated to the ISCO-group Technicians and associate professionals, implying that they have some higher education. In the second level an example would be 32, translated to Life science and health associate professionals. The third level, 323 Nursing associate professionals. And finally, in the fourth level, we have occupations such as 3232 Surgical nurses. We will study the first two levels. We restrict our investigation to these levels because the number of hires that come to pass each month in the more specific levels of occupation are so small, and as the law of large numbers stops applying, statistical problems will arise.

5 All data has been seasonally adjusted using the Tramo/Seats method.
A starting point when connecting our data to the model is getting the right type of unemployment measure, $u_t$. According to the model used, an unemployed person must not work and he or she must be actively searching for a job in a certain sector. We can sort out all the unemployed that are working part time or taking part in programs that are defined as employment. This leaves us with the job-seekers who are without a job and only partaking in labour market programs that are labelled as non-employment; and we know where in Sweden they are registered and what occupation they are looking for. This is a much improved measure, compared to Sahin et al. (2011), seeing as they only know the job-seekers previous sector of work.

The second variable is the hires or matches, $h_t$. In Sahin et al. (2011), they work with data on the total number of hires in a sector, but with our data we can see the number of job-seekers, sorted by our definition above, that left the employment service for employment. Once again, this is very close to the definition in the assumed model.

The last variable we obtain from the employment service is vacancies, $v_t$. The important thing here is that we can sort the vacancies by the same sectors as the job-seekers. When studying Beveridge curves, for example, it is common to use the number of vacancies left at the end of a month to lessen the impact of different random shocks and seasonality effects. This gives a better impression of trends on the labour market, but since we are interested in how many vacancies actually turn into hires in a given month we look at the total amount of registered vacancies each month. If we do not take this into account there is also an issue with certain sectors where the vacancies are posted for a short time, resulting in more hires than vacancies at the end of the month. In other words, we get sectors where there is more than one hire per vacancy, which is mitigated by using the total amount of vacancies.

From Statistics Sweden’s wage statistics we get our proxy for productivity. Much like Sahin et al. (2011), we will use wages as our measure. To avoid issues with very diverse levels of wages, which are not certain to depend on productivity but instead on such things as the strength of labour unions or risk compensation in dangerous jobs etc., we normalise the wage for each occupation around its average and let productivity be the relative change in this variable. Using this method, the wage difference between counties is so small that the productivity variable will have a non-existent effect on the index. Sahin et al. (2011) do not make a point of this, but they do not use productivity in the geographical dimension.

To construct the counterfactual unemployment, which we use to roughly estimate the effect of mismatch inefficiencies on actual unemployment in the Swedish labour market, we need data on unemployment, employment and unemployment duration for the total labour market. This is provided by Statistics Sweden, which recently released linked data sets on unemployment duration back to 1987.
As some of our estimated parameters and variables will be constant over time, it is necessary to choose a point in time from which we can assume that the different variables are somewhat constant. Sweden underwent large structural changes during the nineties with public sector austerity measures and going from a fixed to a floating exchange rate with an inflation target. Assuming that the effects an inflation target will have on the labour market are gradual over time, we need to choose a starting point some years after its implementation. Coinciding with this reasoning are some changes within the occupational definitions in the SSYK-code that were made in 1997 thus lending itself to a good first observation.

As the Swedish Public Employment Service notes whether jobs require previous working experience and whether job-seekers have such experience, it is possible to divide the labour market into two separate entities: one labour market for experienced job-seekers and vacancies demanding such and one labour market for the inexperienced job-seekers, where the vacancies do not state any need for experience. What constitutes experience is presumably not perfectly harmonised between employers and job-seekers, but comparing the two over sectors and time provides us with some interesting results showing definite resemblance to attributes assumed of the youth/non-youth labour markets. As the two labour markets are not completely independent of each other, there might be an issue treating them as two separate markets. For example, it is unlikely that a vacancy demanding experience hires an inexperienced job-seeker, but the other way around should be quite common. The extent to which the latter is common should have a direct effect on matching in both labour markets. It is easy to see that if the vacancies that do not demand experience are taken by experienced job-seekers, this will result in less hires per vacancy in the inexperienced labour market and more hires per vacancy in the experienced. It should also be noted that all vacancies are coded with experience and experience unanimously since 2000. But even before 2000, only a few vacancies were registered without this information. Job-seekers, on the other hand, are more frequent in the lack of information regarding experience. Job-seekers registered without this information range between 3 per cent up to almost 9 per cent at some points.

Even though our data fits the model well, there are still some drawbacks and a need for corrections. Even though there was a law, up until 2007, stating that companies needed to register each vacant position, there seems to be severe underreporting of vacancies in some occupations at specific times. As long as this underreporting is constant over time we will be able to pick up these lost vacancies as improved matching efficiency. If however, for example, the underreporting only occurs in the first half of the studied period and not the second, the increased matching efficiency will be smoothed over the time period, signalling that this sector has a high matching efficiency even in the latter half when the underreporting stopped. To some extent we correct for this by defining vacancies as a variable that must be at least as large as the hires in that sector and time. This will, in other words, increase vacancies in the
correct time period and not smooth it over time, given that there were more hires than vacancies.

Another issue concerns the Swedish Public Employment Service. Their market share varies over the business cycle where there are more reported vacancies in economic upturns. The idea is that employers use more channels when looking for possible employees in upturns as there is more competition with other employers and the need to hire people is larger. This market share variation has been said to be between 30 and 50 per cent (Arbetsförmedlingen 2009). If these fluctuations are symmetric over all sectors, this is not an issue for us, and it will be caught by our time varying matching efficiency \( \Phi_t \), but if different sectors have different reactions to economic variations with respect to their vacancy reporting, it will affect our index to some extent. Sectors that are sensitive to this will have a smaller number of optimal unemployment in downturns than they actually should, and vice versa.

4.2 Stock-flow and random matching models

The choices concerning data are dependent on the attributes assumed of the labour market. An important variable in this choice is whether the matching model describing the labour market is of random or stock-flow types. Random matching implies that all job-seekers have the same probability to find a vacancy and establish a match. Stock-flow matching is based on the assumption that some job-seekers cannot connect to certain vacancies. This implies that new job-seekers are able to match with old vacancies, as opposed to the old job-seekers who have already applied for these vacancies without being able to establish a match and, as such, the old job-seekers can only search for a job amongst the new vacancies. Forslund and Johansson (2007) provide evidence that the Swedish labour market is best described with a stock-flow matching model. In other words, they suggest that there are matching frictions due to heterogeneity amongst both job-seekers and vacancies and that some matches are not possible. However, this does not imply that our choice of data should only follow the stock-flow type.

The choice of data in our case is not self-evident, and it depends on what we want to measure. In this paper, we are hindered from choosing freely between the two, as the data handed to us does not specify if a person hired was previously part of the stock or the inflow. Given that we could have corrected for this there are still some points to be made regarding the choice of data. First of all, our model, from which we derive our index, does not differentiate between the stock of job-seekers and the inflow, it just states that there are a certain number of unemployed in sector \( i \) at time \( t \). This suggests data appropriate for a random matching model. Secondly, if there are matching frictions due to these kinds of stock-flow attributes and if these frictions are diverse in size depending on sector, our goal is to measure the extent of the inefficiencies that these cause. This also suggests data appropriate for the random matching model, as the other type of data corrects for this discrepancy between sectors, thus attenuating the extent of the inefficiency. With that said,
we should take into account that when using the random type data, we will underestimate the elasticity in our matching function, as the stock of unemployed is unlikely to find a job within the stock of vacancies.

Summing up, we can sort the inflow from the stock regarding vacancies and job-seekers, but the hires will always be a combination of the two. Following the reasoning above, we choose to use the stock in time $t-1$ added with the inflow in time $t$ for job-seekers and vacancies, and the hires in time $t$.

4.3 Matching function estimation

To estimate the matching efficiencies we perform panel regressions with time and occupation/region fixed effects on the natural logarithm of hires per unemployed as the dependent and the natural logarithm of vacancies per unemployed as the independent.

$$\ln \left( \frac{h_{it}}{u_{it}} \right) = \ln \Phi_t + \ln \phi_i + \alpha \ln \left( \frac{v_{it}}{u_{it}} \right)$$

The interpretation of the fixed effects in our regression is important. Considering our matching function, $h_{it} = \phi_i \Phi_t v_{it}^a u_{it}^{1-a}$, the time fixed effects will be interpreted as $\ln(\Phi_t)$, the sector fixed effect as $\ln(\phi_i)$ and the coefficient of the independent variable will be the elasticity, $\alpha$.

The matching efficiency, $\phi_i$, will capture all sector-specific effects that are constant over time. The actual value of the matching efficiency will be read as the sector-specific deviation from the geometric mean. If a sector, for example, has a matching efficiency of 1.5, they hire 1.5 times more job-seekers than the geometric mean, given that there is an equal vacancy per job-seeker ratio in each sector. The aggregated matching efficiency $\Phi_t$ will capture all time variation that is constant between sectors. The elasticity, $\alpha$, will thus express the effect a percentage change in labour market tightness has on the percentage change in hires per unemployed where all sector and time fixed effects have been controlled for.
4.4 Descriptive Statistics

Figure 1 V/U and H/U 1997-2011, per cent

The main variables that we analyse are the ratios between vacancies and unemployed, and between hires and unemployed. In figure 1, their development over the last fifteen years, according to our data from the Swedish Public Employment Service, is shown. As can be seen, they trail each other over the years, however, note that the figure has different scales and the hires per unemployed ratio is many times smaller than the vacancy per unemployed ratio. More importantly we note that a change in the vacancy ratio does not have an equal effect on the hires ratio. The elasticities for the aggregated data and our different sectors are estimated in table 1. Compared to the result in Sahin et al. (2011), our aggregated elasticities are quite small. They do not show their estimations for all the different elasticities, but we would assume that our elasticities with fixed effects are smaller than the ones that they estimate. This is to some extent connected to another noticeable attribute of the data, which is the relatively small hires ratio compared to, for example, the job-finding rate that is estimated using data from Statistics Sweden. The job-finding rate should principally describe the same thing, but it does incorporate the total outflow from unemployment, which explains one part of the difference. The other difference is that our hires are not the actual hires in the economy but, instead, the previously unemployed who were registered at the Swedish Public Employment Service and who were hired during the month. However, the job-finding rate and the hires ratio show a similar path over the last 15 years, as can be seen in figure 2.
Table 1 Elasticities

<table>
<thead>
<tr>
<th></th>
<th>Coefficient, $\alpha$</th>
<th>Robust Standard errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregated</td>
<td>0.42***</td>
<td>0.02</td>
</tr>
<tr>
<td>Experienced</td>
<td>0.32***</td>
<td>0.03</td>
</tr>
<tr>
<td>Inexperienced</td>
<td>0.39***</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**Sector and time fixed effects**

<table>
<thead>
<tr>
<th></th>
<th>Coefficient, $\alpha$</th>
<th>Robust Standard errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geographical</td>
<td>0.18***</td>
<td>0.01</td>
</tr>
<tr>
<td>Experienced</td>
<td>0.12***</td>
<td>0.03</td>
</tr>
<tr>
<td>Inexperienced</td>
<td>0.09***</td>
<td>0.01</td>
</tr>
<tr>
<td>Occupational</td>
<td>0.11***</td>
<td>0.02</td>
</tr>
<tr>
<td>Experienced</td>
<td>0.09***</td>
<td>0.02</td>
</tr>
<tr>
<td>Inexperienced</td>
<td>0.08*</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Note: The fixed effect for occupation is based on the more disaggregated occupational code, SSYK2. The quadratic time trend is added to the aggregated data to capture time variation.

Some other noticeable results, studying table 1, are the changes in $\alpha$ depending on whether we are looking at the experienced or inexperienced labour market. That they are consistently smaller than the aggregated case could, at least to some extent, be attributed to the fact that the data on job-seeker experience is less complete than the vacancy data, thus resulting in more vacancies per hire and therefore a smaller $\alpha$. When adding sector and time fixed effects we notice that the estimate of $\alpha$ is reduced for all six variations. A large part of this is time variations, captured by $\Phi_t$. This decrease is greater for the inexperienced workforce, thus showing correlation with a characteristic commonly attributed to the youth-labour force, namely being more sensitive to business cycle fluctuations. Finally, it is evident that both the occupational and geographical fixed effects capture a large part of the influence a change in labour market tightness has on hires, occupational more so than geographical.

**Figure 2 Job-finding rate and hires per unemployed, per cent**

![Graph showing job-finding rate and hires per unemployed](image)

Source: Swedish Public Employment Service and own calculations.

The elasticities that we will use in our index will, much like the ones that Sahin et al. (2011) use, be based on both the estimate from the aggregated data with a
quadratic time trend and on the estimates using sector and time fixed effects. This is done by taking their averages for each dimension and labour market. This stands to reason as our matching function’s $\alpha$ is the same for the aggregated labour market and the different sectors within a given dimension.\(^6\) More specifically, we will for example use an elasticity of 0.3 for the geographical dimension, which does not specify experience, as this is the average of 0.42 and 0.18. As the elasticity expresses the share of vacancies in our matching function, and hence our index, the larger the elasticity the more weight the distribution of vacancies carries, and the smaller, the more importance the sector-specific matching efficiency will carry. In Sahin et al. (2011) the authors do not go into details about their reasoning behind their choice of $\alpha$. One could use a specific $\alpha$ for all dimensions as this simplifies comparability, but as there are such large differences between sectors we choose to use a different $\alpha$ for each dimension and respective experience type. The reasoning is that if we, for instance, note that vacancies have less effect on hires when controlling for occupational effect than for geographical effects, and as such have a larger dispersion across the sector-specific matching efficiencies, this should be noted in both $\alpha$ and $\phi_t$. A smaller $\alpha$ implies an increase in weight for $z_{it}$ and $\phi_t$ in the distribution of job-seekers and with a larger $\alpha$ the actual distribution of vacancies gains importance.

To get a first glimpse of mismatch and an idea of when there is an increase in mismatch, one can study the correlation between the unemployment share and vacancy share in different sectors, as is done in figure 3. If the correlation shrinks, it is an indication that there is a change in matching efficiency within that dimension, if, for example, the vacancy per job-seeker ratio increases without the correlating change in hires per job-seeker, i.e. matching seems to get worse. For counties the correlation is at its lowest point around year 2001, soon after the dotcom-bubble, shortly thereafter the correlation within SSYK2 drops, and it drops again during the financial crisis, as does SSYK1 but to an even greater degree.

---

\(^6\) Aggregated hires are in time $t$ defined as $h_t = \Phi \sum_{i=1}^{n} u_i \left( \sum_{j=1}^{m} \phi_j \left( \frac{u_i}{\phi_j} \right)^2 \right)^{1-\alpha}$. 
5 Empirical Results

Our primary results show that there are efficiency gains to be made by adjusting the allocation of job-seekers. The gains are larger in the occupational dimension than in the geographical, but these results should be interpreted with caution. The occupational mismatch is likely to be more prominent, as this dimension is connected with a higher level of heterogeneity between sectors. Many of these heterogeneities are amongst the job-seekers and, as noted in section 3.5, this is one of the main caveats of the index. So even if the occupational mismatch is greater, the geographical mismatch carries more weight when using the index to describe actual mismatch in the Swedish labour market.

5.1 Measuring geographical mismatch

We will start our mismatch analysis by looking at geographical mismatch. In table 2 each county is represented with its estimated matching efficiency. As can be seen, the most efficient matching, according to our method, is in Dalarna and the least in Kronoberg and Östergötland. Combining these efficiencies with vacancy and unemployment data provides us with our first index.

---

7 We base this solely on the assumption that the average job-seeker displays more similarities amongst each other in different counties than the average job-seeker in different occupations.
### Table 2 Matching efficiency across Swedish counties 1997-2011

<table>
<thead>
<tr>
<th>County</th>
<th>Matching efficiency</th>
<th>Inexperienced</th>
<th>Experienced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dalarna</td>
<td>1.36</td>
<td>1.29</td>
<td>1.37</td>
</tr>
<tr>
<td>Västerbotten</td>
<td>1.17</td>
<td>1.18</td>
<td>1.19</td>
</tr>
<tr>
<td>Gävleborg</td>
<td>1.02</td>
<td>1.01</td>
<td>1.02</td>
</tr>
<tr>
<td>Uppsala</td>
<td>1.06</td>
<td>0.99</td>
<td>1.05</td>
</tr>
<tr>
<td>Västernorrland</td>
<td>0.98</td>
<td>0.95</td>
<td>0.98</td>
</tr>
<tr>
<td>Norrbotten</td>
<td>0.97</td>
<td>0.94</td>
<td>0.92</td>
</tr>
<tr>
<td>Örebro</td>
<td>0.91</td>
<td>0.94</td>
<td>0.92</td>
</tr>
<tr>
<td>Stockholm</td>
<td>0.90</td>
<td>0.92</td>
<td>0.90</td>
</tr>
<tr>
<td>Halland</td>
<td>0.78</td>
<td>0.92</td>
<td>0.86</td>
</tr>
<tr>
<td>Kalmar</td>
<td>0.78</td>
<td>0.92</td>
<td>0.86</td>
</tr>
<tr>
<td>Jönköping</td>
<td>0.78</td>
<td>0.92</td>
<td>0.86</td>
</tr>
<tr>
<td>Gotland</td>
<td>0.78</td>
<td>0.92</td>
<td>0.86</td>
</tr>
<tr>
<td>Skåne</td>
<td>0.78</td>
<td>0.92</td>
<td>0.86</td>
</tr>
<tr>
<td>Västra Götaland</td>
<td>0.78</td>
<td>0.92</td>
<td>0.86</td>
</tr>
<tr>
<td>Västmanland</td>
<td>0.78</td>
<td>0.92</td>
<td>0.86</td>
</tr>
<tr>
<td>Blekinge</td>
<td>0.78</td>
<td>0.92</td>
<td>0.86</td>
</tr>
<tr>
<td>Södermanland</td>
<td>0.78</td>
<td>0.92</td>
<td>0.86</td>
</tr>
<tr>
<td>Östergötland</td>
<td>0.78</td>
<td>0.92</td>
<td>0.86</td>
</tr>
<tr>
<td>Kronoberg</td>
<td>0.78</td>
<td>0.92</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Note: The table shows the matching efficiencies obtained from panel regressions where the logarithm of job-seekers to employment per job-seeker was the independent variable and vacancies per job-seeker was the dependent variable. Counties were treated as group variable and time dummies were used. Matching efficiencies are the group fixed effects for each county and can thus be described as the group-specific deviation from the geometric mean. Uppsala will, for example, hire 1.15 times more job-seekers than the geometric mean, given that there is an equal vacancy per job-seeker ratio in each county.

### Figure 4 Geographic mismatch index

Note: The figure shows the share of matches lost due to mismatch. Source: Swedish Public Employment Service and own calculations.
The geographic mismatch index averages at roughly 0.04 over the whole time period. A noticeable difference, however, is the peak of 0.06 around the dotcom-bubble. The latest point in time that we have data for is November 2011. An index of 0.03 at this point suggests that we can get 3 per cent more hires this month given that we can reallocate the job-seekers to our more efficient allocation. There are gains to be made by reallocation, but we should also look at the distribution of job-seekers suggested by our method and the actual distribution in figure 5. We note that a large amount, approximately 20 per cent, of the unemployed, or at least their job-seeking, needs to be reallocated, primarily to Stockholm, to achieve the 3 per cent increase in job-hiring. Calculating the counterfactual unemployment we note that up to around 0.3 percentage points, on average, could be possible through a more efficient allocation, which can be seen in figure 6. Dividing the market into an experienced and an inexperienced labour market, as in figure 7, we note that the mismatch for the inexperienced is higher during most of the observed time. However, since 2008 the more experienced labour market seems to lose more hires due to mismatch. The distribution of experienced and inexperienced job-seekers at the latest point in time is shown in figure 8.
Figure 6 Counterfactual unemployment, efficient geographical allocation, per cent

Note: The figure shows actual unemployment compared to counterfactual unemployment. Source: Swedish Public Employment Service, Statistics Sweden and own calculations.

Figure 7 Geographical mismatches depending on experience

Source: Swedish Public Employment Service and own calculations.
Figure 8 Efficient geographical distributions, depending on experience, per cent

Note: The figure shows the distribution of experienced and inexperienced job-seekers across counties. As they are different in size and as there are people who do not register as either, none of the distributions in this figure adds up to the one in figure 5.
Source: Swedish Public Employment Service and own calculations.

5.2 Measuring occupational mismatch
If we shift our attention towards occupational mismatch we once again estimate matching efficiencies. The highest matching efficiency is in the Occupational area 7 Craft and related trades workers, and therein, 71 Extraction and building trades workers. This is to some extent due to underreporting of vacancies in this occupational group. The large underreporting and, hence, large matching efficiency, is together with the issues of heterogeneous job-seekers cause for some concern when interpreting the results of the matching index across occupations. When dividing the market into the two experience groups, this is less of an issue for the inexperienced group, which instead is dominated by the occupations in SSYK1’s 1-3 and the subgroups within these for the SSYK2. This can be seen in table 3 and 4.
### Table 3 Matching efficiencies across SSYK1 1997 – 2011

<table>
<thead>
<tr>
<th>Occupational area according to SSYK1</th>
<th>Matching efficiency</th>
<th>Inexperienced</th>
<th>Experienced</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Legislators, senior officials and managers</td>
<td>1.09</td>
<td>1.46</td>
<td>1.04</td>
</tr>
<tr>
<td>2 Professionals</td>
<td>1.07</td>
<td>1.42</td>
<td>0.99</td>
</tr>
<tr>
<td>3 Technicians and associate professionals</td>
<td>0.92</td>
<td>1.20</td>
<td>0.91</td>
</tr>
<tr>
<td>4 Clerks</td>
<td>0.80</td>
<td>0.87</td>
<td>0.78</td>
</tr>
<tr>
<td>5 Service workers and shop and market sales workers</td>
<td>0.67</td>
<td>0.78</td>
<td>0.70</td>
</tr>
<tr>
<td>6 Skilled agricultural and fishery workers</td>
<td>1.58</td>
<td>0.82</td>
<td>1.60</td>
</tr>
<tr>
<td>7 Craft and related trades workers</td>
<td>1.72</td>
<td>1.18</td>
<td>1.68</td>
</tr>
<tr>
<td>8 Plant and machine operators and assemblers</td>
<td>1.12</td>
<td>1.05</td>
<td>1.13</td>
</tr>
<tr>
<td>9 Elementary occupations</td>
<td>0.57</td>
<td>0.58</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Note: See table 2.

### Table 4 Matching efficiencies across SSYK2 1997 – 2011

<table>
<thead>
<tr>
<th>Occupational group according to SSYK2</th>
<th>Matching efficiency</th>
<th>Inexperienced</th>
<th>Experienced</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 Corporate managers</td>
<td>1.15</td>
<td>1.43</td>
<td>1.12</td>
</tr>
<tr>
<td>13 Managers of small enterprises</td>
<td>0.88</td>
<td>1.07</td>
<td>0.84</td>
</tr>
<tr>
<td>21 Physical, mathematical and engineering science professionals</td>
<td>1.16</td>
<td>1.54</td>
<td>1.05</td>
</tr>
<tr>
<td>22 Life science and health professionals</td>
<td>1.12</td>
<td>1.43</td>
<td>1.08</td>
</tr>
<tr>
<td>23 Teaching professionals</td>
<td>1.20</td>
<td>1.44</td>
<td>1.18</td>
</tr>
<tr>
<td>24 Other professionals</td>
<td>1.08</td>
<td>1.35</td>
<td>1.00</td>
</tr>
<tr>
<td>31 Physical and engineering science associate professionals</td>
<td>1.00</td>
<td>1.20</td>
<td>0.95</td>
</tr>
<tr>
<td>32 Life science and health associate professionals</td>
<td>1.19</td>
<td>1.57</td>
<td>1.15</td>
</tr>
<tr>
<td>33 Teaching associate professionals</td>
<td>1.05</td>
<td>1.07</td>
<td>1.07</td>
</tr>
<tr>
<td>34 Other associate professionals</td>
<td>0.93</td>
<td>0.97</td>
<td>0.95</td>
</tr>
<tr>
<td>41 Office clerks</td>
<td>0.75</td>
<td>0.84</td>
<td>0.72</td>
</tr>
<tr>
<td>42 Customer services clerks</td>
<td>0.86</td>
<td>0.93</td>
<td>0.85</td>
</tr>
<tr>
<td>51 Personal and protective services workers</td>
<td>0.71</td>
<td>0.70</td>
<td>0.73</td>
</tr>
<tr>
<td>52 Models, salespersons and demonstrators</td>
<td>0.67</td>
<td>0.72</td>
<td>0.70</td>
</tr>
<tr>
<td>61 Skilled agricultural and fishery workers</td>
<td>1.46</td>
<td>0.80</td>
<td>1.50</td>
</tr>
<tr>
<td>71 Extraction and building trades workers</td>
<td>2.12</td>
<td>1.51</td>
<td>2.12</td>
</tr>
<tr>
<td>72 Metal, machinery and related trades workers</td>
<td>1.29</td>
<td>1.20</td>
<td>1.28</td>
</tr>
<tr>
<td>73 Precision, handicraft, craft printing and related trades workers</td>
<td>0.71</td>
<td>0.78</td>
<td>0.66</td>
</tr>
<tr>
<td>74 Other craft and related trades workers</td>
<td>0.86</td>
<td>0.93</td>
<td>0.81</td>
</tr>
<tr>
<td>81 Stationary-plant and related operators</td>
<td>1.02</td>
<td>1.12</td>
<td>1.05</td>
</tr>
<tr>
<td>82 Machine operators and assemblers</td>
<td>0.90</td>
<td>0.87</td>
<td>0.89</td>
</tr>
<tr>
<td>83 Drivers and mobile-plant operators</td>
<td>1.38</td>
<td>1.18</td>
<td>1.42</td>
</tr>
<tr>
<td>91 Sales and services elementary occupations</td>
<td>0.53</td>
<td>0.51</td>
<td>0.56</td>
</tr>
<tr>
<td>92 Agricultural, fishery and related labourers</td>
<td>1.19</td>
<td>0.47</td>
<td>1.55</td>
</tr>
<tr>
<td>93 Labourers in mining, construction, manufacturing and transport</td>
<td>0.85</td>
<td>0.69</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Note: See table 2.

Source: Swedish Public Employment Service and own calculations.
The main results, seen in figure 9, show an index well above the one describing the geographical mismatch. Seen over the whole time period, the index of SSYK2 has been somewhat more stable than SSYK1 with an index of around 0.18, with a slight increase since 2009, resulting in 0.2 at its most recent point. SSYK1 has been increasing since 2003, except around 2008, coinciding with the high point in Swedish unemployment during the financial crisis, from around 0.07 in late 2002 to more than 0.14, thus somewhat closing the gap between the two levels of aggregation.8 As stated earlier, we cannot answer why there is mismatch, but the composition of the two leaves room for some interpretation. Due to the fact that the two occupational indices are slowly approaching each other, a larger share of the occupational mismatch can be explained by the more aggregated occupational dimension than before. A large part of this is explained by a lack of job-seekers in occupational groups 2 and 3 in SSYK1; the ones that most predominately demand higher education. The matching efficiency and productivity are slightly higher in these groups, but the largest effect on these groups is the increase in number of vacancies over time compared to the other groups.

Figure 9 Occupational mismatch index

Another group that should get more job-seekers, according to our method, is number 7 Craft and related trades workers. This includes the subgroup Extraction and building trades workers, which boasts the highest matching efficiency and thus, in spite of fewer vacancies, is suggested as an occupation that could take in more job-seekers.

As seen in figure 10, the difference between actual and counterfactual unemployment is great. Around 2003, about half of the difference between actual and SSYK2’s counterfactual unemployment was explained by SSYK1-counterfactual unemployment. During the course of time, the discrepancy increases. In November 2011, up to 1.3 percentage point of the unemployment

---

8 It should be noted that a more disaggregated dimension can never have less mismatch than the more aggregated one. In our case this means that the SSYK1 mismatch can, by definition, never be larger than that of SSYK2.
can be explained by mismatch across the occupational areas in SSYK1 and up to about 1.8 point by occupational mismatch in SSYK2.

Figure 10 Unemployment and counterfactual unemployment, SSYK1 and SSYK2, per cent

Note: The figure shows actual unemployment compared to counterfactual unemployment. Source: Swedish Public Employment Service and own calculations.

As we shift our focus towards experience and occupational mismatch, we note in figure 11 that the mismatch is a greater issue for the experienced than for the inexperienced; the mismatch for the SSYK2 averages at around 0.25 and is quite constant, whereas the SSYK1 mismatch is around 0.15, with a drop in mismatch during the financial crisis and afterwards increasing to about 0.18 in November 2011.

The inexperienced show an interesting development over time as the two levels of aggregation converge: SSYK2 falls from 2002 until 2007, without the SSYK1 changing much during this time. From then on, however, both increase at a staggeringly similar pattern and size, suggesting that the majority of the mismatch is due to mismatch across the more aggregated index. The downturn in the SSYK2-index is caused by improvements in allocation within certain occupational groups in SSYK1, primarily within groups 2 and 3 in SSYK1, implying that there is a larger negative change in mismatch within groups 2 and 3 than there is between groups 2 and 3 and the other groups in SSYK1. This can be shown using equation (15) and summing up across the groups, using both data and estimations for SSYK1 and SSYK2. To make eligible figures we sum across SSYK1 groups 1-3, 4-8 and 9, which closely proxies educational level. As we see in figure 12, between 1997 and 2006 there are fewer and fewer erroneously placed job-seekers in groups 1-3 as we sum across SSYK2, which is not evident when we sum across SSYK1, thus proving that the mismatch is within SSYK1 groups 1-3.

9 Groups 1-3 - more than secondary school, 4-8 - Secondary school and 9 - no secondary school demanded.
10 This can of course be done for all occupational groups in SSYK1, which has been done but not included here as the results is evident in figure 12, but doing so it shows that it is predominantly group 2 and 3 that shows this development.
Since 2006 a larger share of vacancies are within occupational groups 1, 2 and 3 and this coincides with comparatively less job-seeking being done in these groups and, therefore, increasing both indices.

**Figure 12 Share of inefficiently placed job-seekers**

Note: The figure shows the share of inefficiently placed job-seekers within different occupational groups. During 2007-2008 the share is larger for the SSYK2 level of disaggregation, which is due to \( \frac{X_{it}}{X_{it}} \) as this only follows the increase in disaggregation property on average over the whole period in time and not for each specific point in time.

Source: Swedish Public Employment Service and own calculations.
6 Discussion

Interpreting the results must be done with caution as the indices and counterfactual unemployment come with certain limitations. As noted in section 3.5, a main issue with the mismatch index is that it captures all matching frictions, even immeasurable attributes of job-seekers hindering assumptions of first best allocation. Arguably this problem should be of less concern regarding the geographical index, as average job-seekers presumably are more similar across counties than occupations. The issue is also lessened as we look at different levels of experience, as this should (at least to some extent) capture job-seeker heterogeneity. Combining these two assumptions provides us with reason to assume that the geographical dimension is less hampered by job-seeker heterogeneity. As the two indices for experienced and inexperienced are quite similar in the geographical dimension, while the same indices in the occupational dimension vary greatly, it shows that the occupational experience/inexperience indices capture a lot more job-seeker heterogeneity and the results in this dimension should be interpreted with more caution than the others.

6.1 Interpreting the results

There are inefficiencies in the Swedish labour market in both of the studied dimensions. The index for the geographical mismatch averages at around 0.03 the last few years and the counterfactual unemployment over that period is approximately 0.3 percentage points less than the actual. To achieve this more efficient allocation, it would be necessary to move approximately 20 per cent of the job-seekers, which, needless to say, is a lot of people. Also, our measure is an upper bound of mismatch, as it captures all frictions and does not incorporate allocation costs. It also does not take into account further dynamic effects that should be relevant, especially in the geographical dimension. As our index shows, more people should move to primarily Stockholm; besides the increase in job-finding probability, this should also have many long-term positive effects on consumption, investment and the like. So, assuming that a job-seeker in Stockholm is somewhat similar to other job-seekers across Sweden, there are beneficial effects of reallocating people’s search efforts towards Stockholm. This could be done through, for example, policies that would ease moving to Stockholm, an efficient housing market and increased possibilities to commute to the capital.

The indices for the occupational dimension are all quite large and growing, but even so, the interpretation and policy relevance must be done with caution as this dimension, to a much higher degree, is associated with different types of inefficiencies that are impossible to correct for. Their size relative to the geographical index is difficult to judge, but their development over time is cause for some concern. Comparing the inexperienced and experienced labour market, we note that the inexperienced labour market is less affected by mismatch in terms of pure index size.\(^{11}\) Even though mismatch was less of an

\(^{11}\) This can be related to other studies that have shown that the job-finding rate and labour market flexibility is higher when considering the young (Swedish Fiscal Policy Council, 2009).
issue during the financial crisis, according to our index\(^{12}\), the occupational mismatch indices have all grown and are at high points compared to the average. As we have seen, the more aggregated versions are closing in on the more disaggregated, giving us a hint of what is happening. This change is most apparent when studying the inexperienced labour market over the occupational dimension where there is a definite change in composition. We do not know why this change occurs and whether it depends on imperfect information, educational cost, or different hiring processes, but we can state that there are increasing vacancy shares among occupations that require higher education, which are not matched with high enough increases in job-seeker shares. Our findings suggest that there are efficiency gains to be captured in reallocating inexperienced job-seekers to occupations demanding more of an education, without taking into account the deeper causes or means for how to implement this.

### 6.2 Comparisons with previous studies

As the methods used in this paper are relatively new there is a lack of international studies to compare our results with. So far the only comparable results are in Sahin et al. (2011). Similar to our result, they show that the geographical mismatch affects unemployment less than industrial and occupational mismatch. Occupational and industrial mismatch is thought to explain between 0.6 and 1.7 percentage points of the increase in unemployment, from the start of the recession to the end of 2009. Another point they make is that the role of mismatch varies between different levels of education. Somewhat similar to our results regarding the inexperienced workforce, they show that less educated workers are quantitatively less affected by mismatch compared to more educated workers.

A main reason as to why they get smaller indices is that they estimate larger elasticities, which lessens the impact of the matching efficiencies when constructing the index. To some extent, America is thought to have a higher job-finding rate, and thus a larger elasticity with this method, but we can also attribute our small elasticity to a lack of stock-flow model appropriate data. Furthermore, we are not looking at the economy as a whole but instead the people registered at the Swedish Public Employment Service, which reports quite few hires per month compared to the total outflow from unemployment. Nevertheless, it is intriguing that they get a smaller index for the geographical dimension than we do, even though they study the whole of the United States, which includes more sectors, fitting some classic notions that the American labour market is more flexible than its European counterparts. Their upper bound for the occupational index is similar to ours, but it should be noted that they are studying occupations (which would be equivalent of SSYK3), suggesting that our results, in reality, are higher here as well.

---

\(^{12}\) This fits well with modern analysis of labour market dynamics. Michaillat (2012), for instance, shows that search friction is of less concern in recessions: as jobs are lacking, each vacant job is filled rapidly and at low cost, in spite of matching frictions. This results in lower marginal cost of labour as the recruiting expenses fall, thus decreasing the frictional unemployment. Michaillat instead argues that job-rationing explains the higher levels of unemployment during downturns.
Looking towards similar Swedish matching studies, they are far and few between. Aranki and Löf (2009) show some interesting results regarding the geographical matching efficiencies across Swedish counties. Somewhat similar to our results, they argue that the densely populated areas have less matching efficiency. Our densely populated counties, on average, have less matching efficiencies than the country as whole, much due to Västra Götaland and Skåne showing lower matching efficiencies, like in Aranki and Löf (2009). However, in contrast to our results, they find that Stockholm also has a low matching efficiency. In our estimations Stockholm has an average matching efficiency, and due to the large share of vacancies in Stockholm, people should still seek jobs there. Part of the difference could be explained by the method, as their goal is to study the labour market by estimating a matching function based on stock-flow assumptions. We do not set out to fit a matching function but instead attempt to capture matching frictions, even those caused by the stock of unemployed not finding work within the stock of vacancies, unlike Aranki and Löf (2009). Our different results might therefore be explained by stocks of job-seekers being unevenly distributed amongst Swedish counties and the low probability of getting a job from both the stock and inflow of vacancies. This is treated as a search friction by us, and something we want to compare amongst counties, whilst Aranki and Löf (2009) set out to circumvent it and thus appropriately estimate a matching function that more accurately describes how changes in vacancies and unemployment will affect matches.

Another difference is that they look further back in time, something we choose not to do, as we use constant sector fixed effects which cannot be assumed to be constant across time given the structural changes during the nineties. They use regional dummies as well, but they do not use these for anything else than to control for regional differences that are constant over time.

6.3 Further studies

A main point to make yet again, as we acknowledge the need for further studies, is that the method in this paper does not attempt to answer why there is mismatch and frictions in the labour market, but instead only serves as a first indication to where the inefficiencies are most prominent and how these develop over time. Developing and solving an equilibrium model where all possible frictions are accounted for seems far off, but there are other choices and adjustments of data within the method that can be done so as to improve our understanding of labour market frictions.

First of all, obtaining data where one can see if a job-seeker who gets hired was in the stock or in the inflow that period would give us the opportunity to correct for the errors of not using stock-flow-appropriate data. Comparing our results with these would be an interesting next step, as we would get more appropriate elasticities and the difference between our results and future results could show the extent of the matching frictions assumed in the stock-flow models due to job-seeker heterogeneity.
In general, it would be interesting to gather all possible data which can be sorted into vacancies, unemployed and matches. Possible and interesting data would be across the industry dimension or other types of educational data.

The method is most appropriately used as a first step in analysing the labour market; showing where the issues are most grave and what the changes are over time. Using this method to more accurately pinpoint where further studies are needed is a central contribution of this paper. How to optimally use this method in this line of work is in itself a warrant for future studies and should be cause of some contemplation.

7 Conclusions

In this paper we have created a measurement for labour market mismatch, defined as the difference between an efficient allocation of job-seekers and the actual distribution of job-seekers. This was done by deriving an allocation rule from a dynamic search model where a social planner was to choose the optimal allocation of job-seekers. This rule was then used to create an index of mismatch.

By using our index we demonstrated that there is a geographical mismatch in the Swedish labour market explaining up to 0.3 percentage points of the current unemployment. For the inexperienced and experienced labour markets, the geographical mismatch indices show similar patterns over time. A slight decrease for the inexperienced labour market results in it being smaller than the mismatch for the experienced at the most current points in time, contrary to earlier points in data. As the geographical mismatch index provides us with more conclusive results than the occupational index, this suggests that reallocation of job-seekers should at least warrant some further attention, as there seem to be gains to be made through policy choices taking this into account.

We also showed by using our method that there is a large, but difficult to interpret, occupational mismatch in the Swedish labour market, which could explain up to 1.8 per cent of the unemployment. We cannot answer why this mismatch exists, but through studying its trend and different levels of disaggregation we conclude that it has been changing in size and composition over the last few years, especially for the inexperienced. Furthering our study of occupational mismatch we note that the mismatch index is less for the inexperienced job-seekers at the Swedish Public Employment Service. This is in line with other studies which show that the youth labour force is more dynamic and flexible and that their job-finding rate is higher.

Lastly, we see further uses for the methods presented in this paper, as they have served well in this first approach to analysing labour-market matching.
References
Swedish Fiscal Policy Council (2009), Swedish fiscal policy, Finanspolitiska rådets rapport 2009.
Appendix A – Solving the planner’s problem

With the choices of unemployment allocation in this period and labour force size in the next period we want to maximise the following function.

\[ V(e, u; v, \phi, z, Z, \delta, \Phi) = \max_{\{u_i, l\}'} \sum_{i=1}^{I} Z_{zi}(e_i + \gamma h_i) - \xi u + \beta \mathbb{E}[V(e', u'; v', \phi', z', Z', \delta', \Phi')] \]

subject to

\[ \sum_{i=1}^{I} u_i \leq u \]  \hspace{1cm} (1)

\[ h_i = \Phi \phi_i m_i(u_i, v) \]  \hspace{1cm} (2)

\[ e_i' = (1 - \delta)(e_i + h_i) \]  \hspace{1cm} (3)

\[ u' = l' - \sum_{i=1}^{I} e_i' \]  \hspace{1cm} (4)

\[ u_i \in [0, u] , \quad l' \in [0, 1] \]  \hspace{1cm} (5)

\[ \Gamma_{Z, \delta, \phi}(Z', \delta', \Phi'; Z, \delta, \Phi), \Gamma_{v}(v' ; v, Z', \delta', \Phi'), \Gamma_{\phi}(\phi'; \phi), \Gamma_{z}(z'; z) \]  \hspace{1cm} (6)

The choice of unemployment distribution across sectors implies a first order condition, which we get by specifying the following Lagrange function where \( \mu \) is the Lagrange multiplier which can be interpreted as the shadow value of one more unemployed person in the stock.

\[ L(u_i) = \sum_{i=1}^{I} Z_{zi}(e_i + \gamma h_i) - \xi u + \beta \mathbb{E}[V(e', u'; v', \phi', z', Z', \delta', \Phi')] - \mu(u_i - u) \]

Taking the derivative of \( L(u_i) \) with respect to \( u_i \) and plugging in restriction (3) and (4) we get the following expression.

\[ \frac{\partial L(u_i)}{\partial u_i} = Z_{zi} \gamma \Phi_i m_i(u_i) \left( \frac{v_i}{u_i} \right) + (1 - \delta) \Phi_i m_i(u_i) \left( \frac{v_i}{u_i} \right) \beta \mathbb{E} \left[ -V_u(\cdot) + V_{u'}(\cdot) \right] - \mu = 0 \]

\[ Z_{zi} \gamma \Phi_i m_i u_i \left( \frac{v_i}{u_i} \right) + (1 - \delta) \Phi_i m_i u_i \left( \frac{v_i}{u_i} \right) \beta \mathbb{E} \left[ -V_u(\cdot) + V_{u'}(\cdot) \right] = \mu \]  \hspace{1cm} (7)

Equation (7) states that the shadow value of one more unemployed in the stock equals the expected value of one additional unemployed person seeking work in sector \( i \), which the left hand side can be interpreted as.
The marginal value of employment in sector \( i \) is obtained by using the envelope theorem with respect to \( e_i \):

\[
V_{e_i}(e, u; v, \phi, z, Z, \delta, \Phi) = \frac{\partial L}{\partial e_i} = zz_i + \beta (1 - \delta) \mathbb{E}[V'_{e_i}(\cdot) - V''_{e_i}(\cdot)]
\]  \hspace{1cm} (8)

The marginal value of another unemployed person in the stock we get by using the envelop theorem with respect to \( u \):

\[
V_u(e, u; v, \phi, z, Z, \delta, \Phi) = \frac{\partial L}{\partial u} = \mu - \xi
\]  \hspace{1cm} (9)

The marginal value of an additional employee (8) is the flow output in sector \( i \) plus the discounted expected value of employment in the next period minus the value of search in the next period. The marginal utility of an additional unemployed person in the stock (9) is the shadow value of another unemployed person in the stock minus the negative utility of job search.

The first order condition of the planner's problem with respect to next period's labour force gives us:

\[
\frac{\partial L(l')}{\partial l'} = \beta \mathbb{E}[V_u(e', u'; v', \phi', z', Z', \delta', \Phi')] = 0
\]  \hspace{1cm} (10)

The marginal value of an additional person in the labour force should be the same as the expected value of not participating in the labour force in the optimal case. If we assume that the expected value of non-participation in the labour force is zero, it coincides with equation (10) which shows that the expected value of one more in the labour force is equal to zero when optimising next period's size of the labour force. Combining equation (9) and (10), we show that the expected shadow value must be equal to the negative utility of seeking a job.

\[
\mathbb{E}[V_u(e', u'; v', \phi', z', Z', \delta', \Phi')] = \mathbb{E}[\mu'] - \xi = 0
\]  \hspace{1cm} (11)

To be able to use the sector-specific productivity we must make the following conjecture:

\[
V_{ei}(e, u; v, \phi, z, Z, \delta, \Phi) = z_i \psi(Z, \delta, \Phi)
\]  \hspace{1cm} (12)

Which states that the marginal value of employing in sector \( i \) can be described as the sector-specific productivity multiplied with a function that only depends on the stochastic variables \( Z, \delta \) and \( \Phi \). To see if it holds, we can plug in equation (12) into (8) and remind ourselves that the expected marginal value of one more job-seeker is zero.
\[ z_i \psi(Z, \delta, \Phi) = Z z_i + \beta (1 - \delta) \mathbb{E}[z_i \psi(Z', \delta', \Phi')] \]

As we have assumed that \( z_i \) has a linear conditional mean function we can rewrite the above equation as

\[ z_i \psi(Z, \delta, \Phi) = Z z_i + \rho z_i \beta (1 - \delta) \mathbb{E}[\psi(Z', \delta', \Phi')] \]

This we divide with \( z_i \) and get

\[ \psi(Z, \delta, \Phi) = Z + \rho \beta (1 - \delta) \mathbb{E}[\psi(Z', \delta', \Phi')] \] (13)

which confirms our conjecture.

By plugging in equation (13) into (7) we get

\[
Z z_i \gamma \Phi i m u_i \left( \frac{v_i}{u_i} \right) + (1 - \delta) \Phi i m u_i \left( \frac{v_i}{u_i} \right) \beta \rho z_i \mathbb{E}[\psi(Z', \delta', \Phi')] = \mu
\]

\[
z_i \Phi_i m u_i \left( \frac{v_i}{u_i} \right) (Z \gamma \Phi + (1 - \delta) \Phi \beta \rho \mathbb{E}[\psi(Z', \delta', \Phi')]) = \mu
\]

\[ z_i \Phi_i m u_i \left( \frac{v_i}{u_i} \right) = \frac{\mu}{(Z \gamma \Phi + (1 - \delta) \Phi \beta \rho \mathbb{E}[\psi(Z', \delta', \Phi')])} \]

which applies to all sectors, seeing as the right hand side is independent of sector.

If we rewrite the optimal number of unemployed in sector \( i \) as \( u_i^* \) we can express the planner’s allocation rule (14)

\[
z_i \Phi_i m u_i \left( \frac{v_i}{u_i} \right) = \cdots = z_i \Phi_i m u_i \left( \frac{v_i}{u_i} \right) = \cdots = z_i \Phi_i m u_i \left( \frac{v_i}{u_i} \right) \] (14)

This explains how the unemployed should be distributed across sectors. A larger (smaller) ratio between vacancies and unemployed, a higher (lower) matching efficiency and a higher (lower) productivity, suggests that there should be more (less) unemployed people in that sector.

From equation (14) we get the optimal distribution of unemployed. By combining this with our matching function we can derive the optimal amount of unemployed in sector \( i \). Our matching function is defined as

\[ m(v_i, u_i) = v_i^{\alpha} u_i^{1-\alpha} \] (15)
Plugging in (15) into (14) we get

\[ z_1 \phi_1 \left( \frac{v_1}{u_1} \right)^\alpha = \cdots = z_i \phi_i \left( \frac{v_i}{u_i} \right)^\alpha = \cdots = z_L \phi_L \left( \frac{v_L}{u_L} \right)^\alpha \]  

(16)

Assuming two sectors, \( i \) and \( j \), we are left with

\[ z_i \phi_i \left( \frac{v_{it}}{u_{it}^*} \right)^\alpha = z_j \phi_j \left( \frac{v_{jt}}{u_{jt}^*} \right)^\alpha \]

which gives us the following expression for \( u_{it}^* \)

\[ u_{it}^* = \frac{v_{it}}{v_{jt}} \cdot \frac{u_{jt}^*}{u_{it}^*} \cdot \left( \frac{z_i \phi_i}{z_j \phi_j} \right)^{\frac{1}{\alpha}} \]

Summing across \( j \)

\[ \sum_{j=1}^{J} u_{jt}^* = u_t = \sum_{j=1}^{J} \frac{v_{jt}}{v_{it}} \cdot u_{jt}^* \cdot \frac{1}{\left( z_{jt} \phi_j \right)^{\frac{1}{\alpha}}} = \sum_{j=1}^{J} \left( z_{jt} \phi_j \right)^{\frac{1}{\alpha}} \cdot \frac{u_{jt}^*}{v_{jt} \cdot \left( z_{jt} \phi_j \right)^{\frac{1}{\alpha}}} \]

Rearranging and noting that \( \sum_{i=1}^{I} \left( v_{it} \cdot \left( z_{it} \phi_i \right)^{\frac{1}{\alpha}} \right) = \sum_{j=1}^{J} \left( v_{jt} \cdot \left( z_{jt} \phi_j \right)^{\frac{1}{\alpha}} \right) \) we get a general expression for the optimal unemployment in sector \( i \) at time \( t \).

\[ u_{it}^* = \left( z_{it} \phi_i \right)^{\frac{1}{\alpha}} \cdot \frac{v_{it}}{\sum_{i=1}^{I} \left( v_{it} \cdot \left( z_{it} \phi_i \right)^{\frac{1}{\alpha}} \right)} \cdot u_t \]

(17)
Appendix B – Deriving the mismatch index

The optimal amount of hires is expressed as

\[ h_t^* = \Phi_t v_t^\alpha u_t^{1-\alpha} \left[ \sum_{i=1}^I \phi_i \left( \frac{v_{it}}{v_t} \right)^\alpha \left( \frac{u_t}{u_t} \right)^{1-\alpha} \right] \]  \hspace{1cm} (13) \]

which together with equation (10) gives us

\[ h_t^* = \Phi_t v_t^\alpha u_t^{1-\alpha} \left[ \sum_{i=1}^I \phi_i \left( \frac{v_{it}}{v_t} \right)^\alpha \left( \frac{v_{it}}{\Sigma_{i=1}^I v_{it}(\zeta_{it})^\alpha} \right)^{1-\alpha} \right] \]

To simplify our notation we will from now on write \( z_{it} \Phi_{it} = x_{it} \) which is a measure of the total matching efficiency in sector \( i \) at time \( t \). We now multiply with \( \left( \frac{x_{it}}{\zeta_{it}} \right)^\frac{1}{\alpha} \) giving us

\[ h_t^* = \Phi_t v_t^\alpha u_t^{1-\alpha} \left[ \sum_{i=1}^I \phi_i \left( \frac{v_{it}(x_{it})^\frac{1}{\alpha}}{v_t} \right)^\alpha \left( \frac{1}{(x_{it})^\frac{1}{\alpha}} \right)^\alpha \left( \frac{v_{it}}{\Sigma_{i=1}^I (v_{it}(x_{it})^\frac{1}{\alpha})} \right) \right] \]

\( \left( \sum_{i=1}^I x_{it}^\frac{1}{\alpha} \frac{v_{it}}{v_t} \right)^\alpha \) can be interpreted as the market-level overall efficiencies weighted by each sectors vacancy share. We will denote it \( \bar{x}_t \).

\[ h_t^* = \Phi_t v_t^\alpha u_t^{1-\alpha} \left[ \bar{x}_t \sum_{i=1}^I \left( \frac{1}{\zeta_{it}} \right) \left( \frac{v_{it}}{\Sigma_{i=1}^I (v_{it}(x_{it})^\frac{1}{\alpha})} \right) \right] \]

We multiply by \( \frac{E_t}{v_t} \) giving us an expression with vacancy shares

\[ h_t^* = \Phi_t v_t^\alpha u_t^{1-\alpha} \left[ \bar{x}_t \cdot \frac{\sum_{i=1}^I \left( \frac{1}{\zeta_{it}} \right) x_{it}^\frac{1}{\alpha} \left( \frac{v_{it}}{v_t} \right)}{\Sigma_{i=1}^I x_{it}^\frac{1}{\alpha} \left( \frac{v_{it}}{v_t} \right)} \right] \]

where we do the following connotation

\[ \bar{x}_t \cdot \frac{\sum_{i=1}^I \left( \frac{1}{\zeta_{it}} \right) x_{it}^\frac{1}{\alpha} \left( \frac{v_{it}}{v_t} \right)}{\Sigma_{i=1}^I x_{it}^\frac{1}{\alpha} \left( \frac{v_{it}}{v_t} \right)} = \bar{\phi}_{xt} \]
which we use to describe optimal hirings as

$$h_t^* = \Phi_t v_t^a u_t^{1-a} \tilde{\phi}_{xt}$$

And since actual hirings are described as,

$$h_t = \Phi_t v_t^a u_t^{1-a} \left[ \sum_{i=1}^{l} \phi_i \left( \frac{v_i}{v_t} \right)^a \left( \frac{u_i}{u_t} \right)^{1-a} \right],$$

it shows that actual matches are not directly affected by productivity, which the planner instead takes into account. The index will be

$$M_{xt}^p = 1 - \frac{h_t}{h_t^*} = 1 - \frac{\Phi_t v_t^a u_t^{1-a} \left[ \sum_{i=1}^{l} \phi_i \left( \frac{v_i}{v_t} \right)^a \left( \frac{u_i}{u_t} \right)^{1-a} \right]}{\Phi_t v_t^a u_t^{1-a} \tilde{\phi}_{xt}}$$

$$= 1 - \sum_{i=1}^{l} \left( \frac{\phi_i}{\tilde{\phi}_{xt}} \right) \left( \frac{v_i}{v_t} \right)^a \left( \frac{u_i}{u_t} \right)^{1-a}$$

We can also derive the amount of wrongly placed job-seekers

$$u_t^M = \frac{1}{2} \sum_{i=1}^{l} |u_{it} - u_{it}^*|$$

We substitute our previously obtained $u_{it}^*$

$$u_t^M = \frac{1}{2} \sum_{i=1}^{l} \left| u_{it} - \frac{1}{\sum_{i=1}^{l} \left( \frac{v_{it} x_i^{a}}{v_t} \right)^{1-a}} \frac{v_t}{v_t} u_t \right|$$

which we then divide with $u_t$ and which thus provides us with a measurement of the share of misallocated job-seekers.

$$\frac{u_t^M}{u_t} = \frac{1}{2} \sum_{i=1}^{l} \left| \frac{u_{it}}{u_t} - \frac{1}{\frac{x_i^{a}}{x_t} \frac{v_{it}}{v_t}} \right|$$
Studier i finanspolitik

2008/1 Alan Auerbach: Long-term objectives for government debt.

2008/2 Roel Beetsma: A survey of the effects of discretionary fiscal policy.

2008/3 Frederick van der Ploeg: Structural reforms, public investment and the fiscal stance: a prudent approach.


2008/5 Per Molander och Gert Paulsson: Vidareutveckling av det finanspolitiska regelverket.


2008/7 Ann Öberg: Incitamentseffekter av slopad fastighetsskatt.


2009/5 Per Molander: Net wealth analysis and long-term fiscal policymaking.

2009/6 Oskar Nordström Skans: Varför är den svenska ungdomsarbetslösheten så hög?

2009/7 Gabriella Sjögren Lindquist och Eskil Wadensjö: Arbetsmarknaden för de äldre.

2010/1 Michael Bergman: Hur varaktig är en förändring i arbetslösheten?


2010/5 Pathric Hägglund och Peter Skogman Thoursie: De senaste reformerna inom sjukförsäkringen: En diskussion om deras förväntade effekter.
2010/6 Christopher A Pissarides: Regular Education as a Tool of Countercyclical Employment Policy.

2010/7 Per Skedinger: Hur fungerar arbetsmarknadspolitiken under olika konjunkturlägen?

2010/8 Lars Calmfors: Fiscal Policy Coordination in Europe.

2010/9 Lars Calmfors: The role of independent fiscal policy institutions.


2011/3 Peter Fredriksson och Jonas Vlachos: Reformen och resultat: Kommer regeringens utbildningsreformer att ha någon betydelse?


2012/2 Göran Hjelm och Ulla Robling: Utvecklingen av de offentliga finanserna till 2020 vid fem olika makroekonomiska scenarier.


2012/4 Jesper Roine: Varför ska vi bry oss om fördelningsfrågor?

2012/5 Gabriella Sjögren Lindquist och Eskil Wadensjö: Inkomstfördelningen bland pensionärer.